An Adaptive Multiscale Numerical Method for Highly Nonlinear Internal Waves

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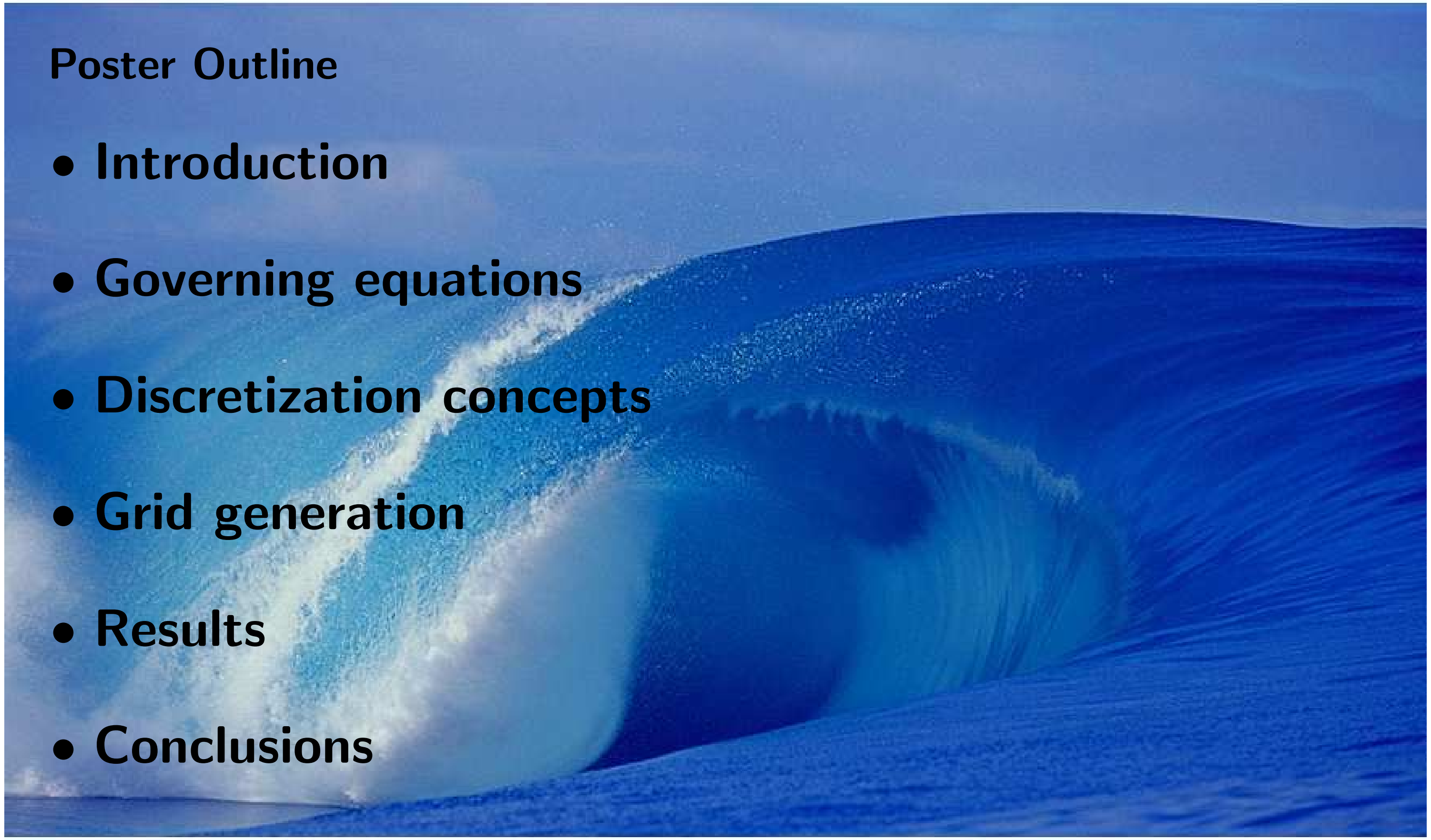


Poster Outline

- **Introduction**
- **Governing equations**
- **Discretization concepts**
- **Grid generation**
- **Results**
- **Conclusions**

Introduction

What are the key issues for modeling multiscale highly nonlinear internal waves?

- Need to capture generation, propagation, and dissipation
- Simplified equation sets won't work, need to solve incompressible Navier-Stokes equations
- Large ranges in spatial and temporal scales
- Internal waves interact with complex bathymetry

What do we hope to provide with this method?

- An enhanced ability to interpret and extend the results of field and laboratory studies
- A predictive tool for both engineering and science

Variable Density Incompressible Navier-Stokes Equations

- Momentum balance

$$\vec{u}_t + (\vec{u} \cdot \nabla)\vec{u} = -\frac{\nabla p}{\rho} + \vec{g} + \nu \Delta \vec{u}$$

- Divergence free constraint

$$\nabla \cdot \vec{u} = 0$$

- Density conservation

$$\rho_t + \vec{u} \cdot \nabla \rho = 0$$

- Passive scalar transport

$$c_t + \vec{u} \cdot \nabla c = \nabla \cdot (k_c \nabla c) + H_c$$

Note that we do not employ Boussinesq or hydrostatic approximations.

Solution Strategy: Temporal Discretization

We build on a classic second-order accurate projection method (Bell, Colella, Glaz, JCP 1989). We split the momentum equations into three pieces:

- Hyperbolic: $\vec{u}_t + (\vec{u} \cdot \nabla)\vec{u} = H$

  where we exactly enforce a divergence free state for the advective velocities, and compute the advective term explicitly

- Parabolic: $\vec{u}_t = \nu \Delta \vec{u} + S$

  which we solve implicitly for a predictor velocity

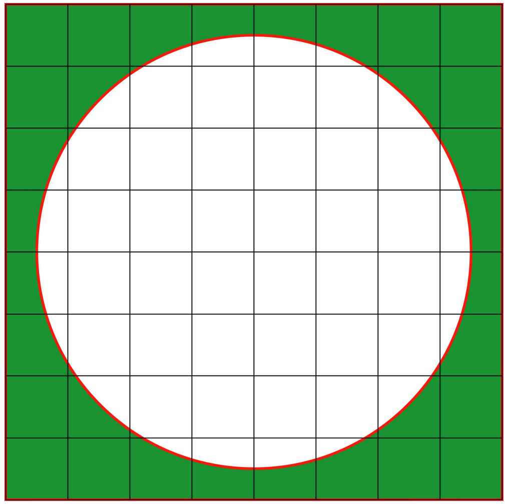
- Elliptic: $\nabla \cdot \frac{1}{\rho} \nabla p = \nabla \cdot (-(\vec{u} \cdot \nabla)\vec{u} + \nu \Delta \vec{u})$

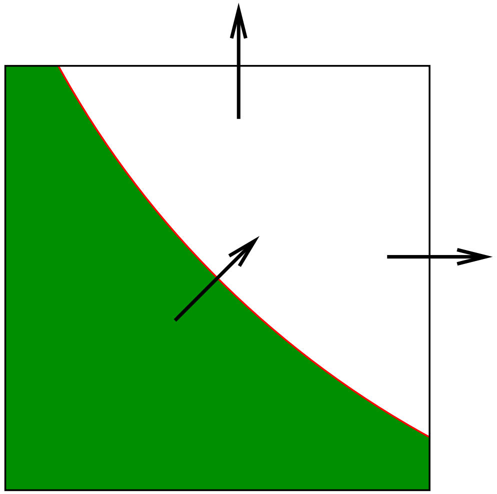
  which we solve implicitly for pressure, and subsequently correct the predictor velocity

To update the scalar equations we do similar hyperbolic and parabolic decompositions.

Solution Strategy: Spatial Discretization Using Embedded Boundaries (EB)

For the bulk of the flow, $O(n^3)$ cells in 3D, we compute on a regular Cartesian grid. We use an embedded boundary description for the $O(n^2)$ control-volumes (in 3D) that intersect the boundary.





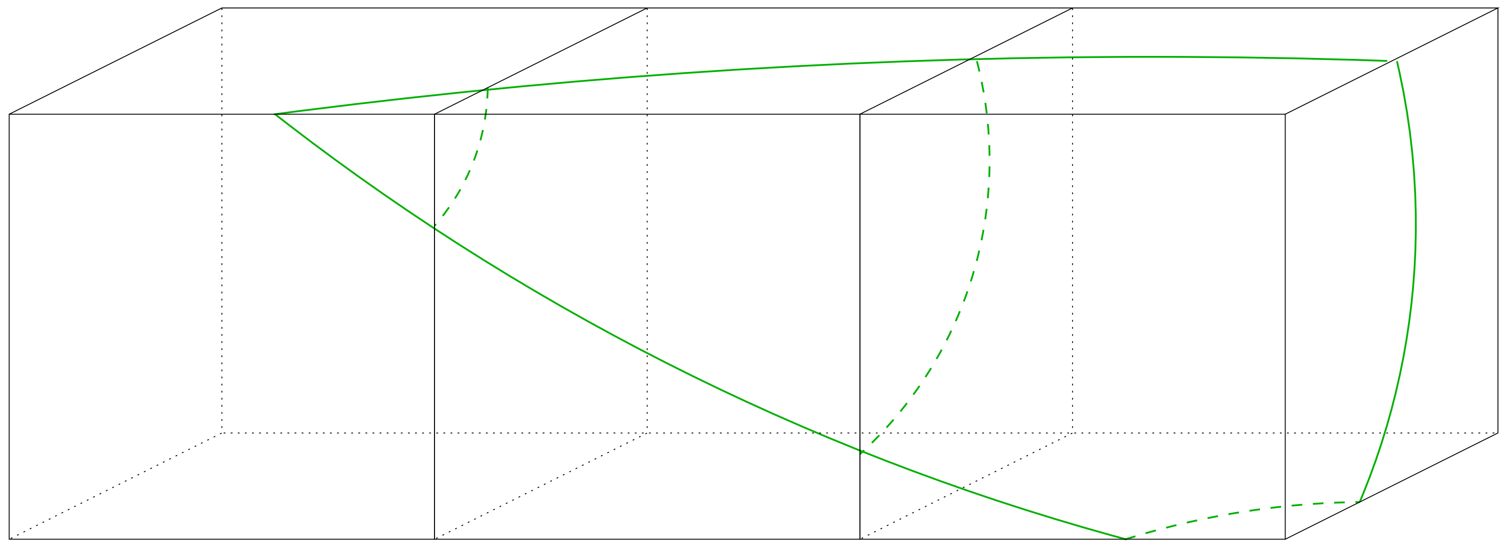
Advantages of underlying rectangular grid:

• Grid generation is tractable, with a straightforward coupling to block-structured adaptive mesh refinement (AMR)

• Good discretization technology, e.g. well-understood consistency theory for finite differences, geometric multigrid for elliptic solvers.

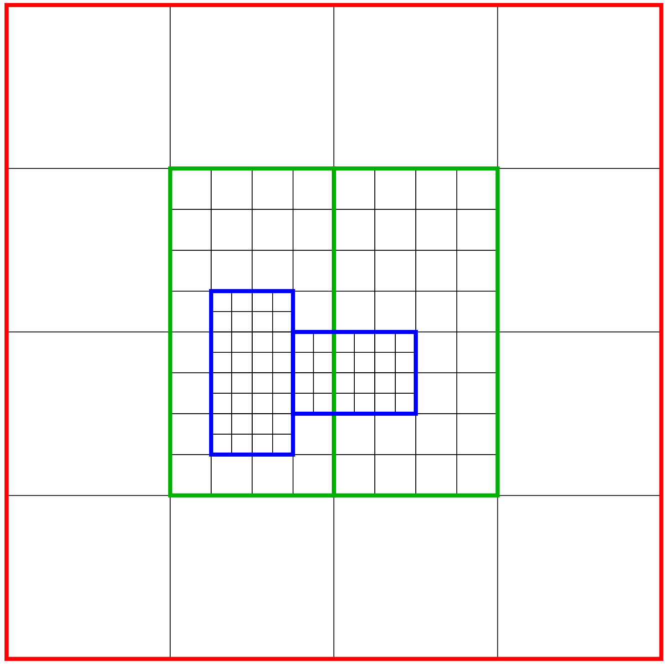
Embedded Boundary Control Volumes

Three example irregular cells are shown below. Green curves indicate the intersection of the exact boundary with a Cartesian cell. We approximate face intersections using quadratic interpolants.



- For each control volume we compute: volume fractions, area fractions, centroids, boundary areas, and boundary normals. These are all we need for discretizing our conservation laws.
- EB's are second-order accurate. Stair-step methods are first-order accurate for area error, and zero-order accurate for perimeter and boundary normal errors.

Block-Structured Adaptive Mesh Refinement

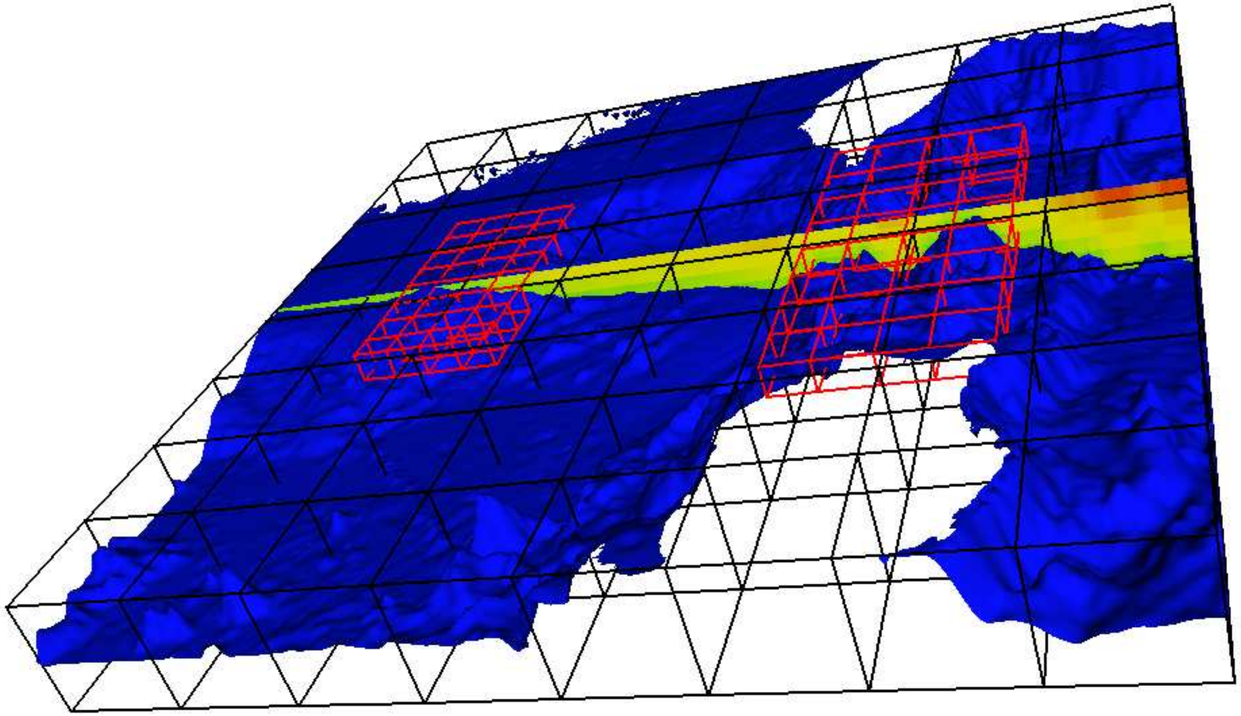


In adaptive methods, one adjusts the computational effort locally to maintain a uniform level of accuracy throughout the problem domain.

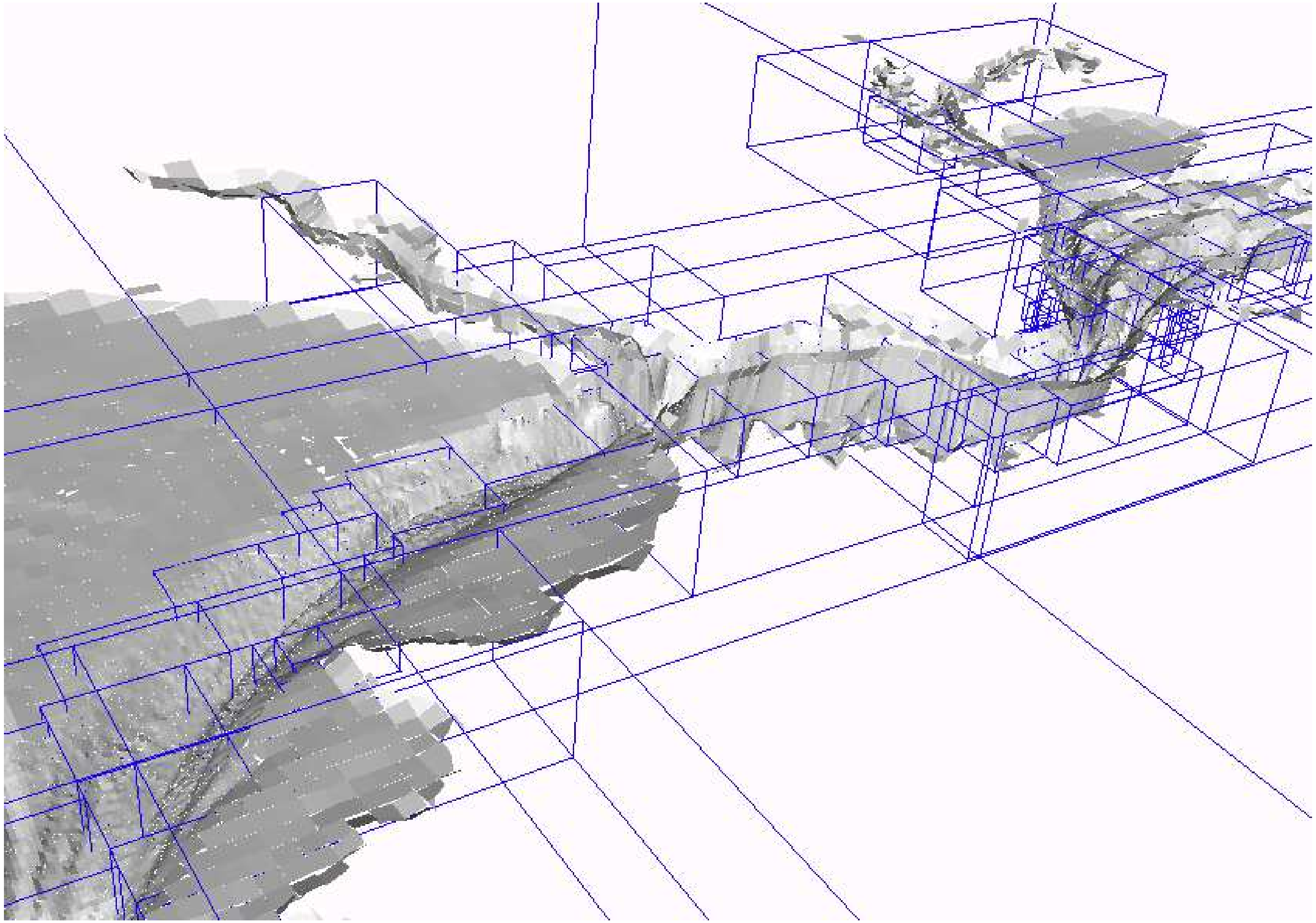
- Refined regions are organized into rectangular patches. Refinement is possible in both space and time.
- Using EB AMR finite-volume methods we maintain conservation and second-order accuracy.

EB AMR Grid Generation Examples: South China Sea

- Black boxes are a decomposition of the coarsest level, red boxes are finer grids. Each box is further sub-divided into individual control volumes. The red boxes refine both the Luzon Strait vicinity (right) and the Dongsha Island region (left). Upper right is Taiwan, lower right is the Philippines, mainland China is upper left.



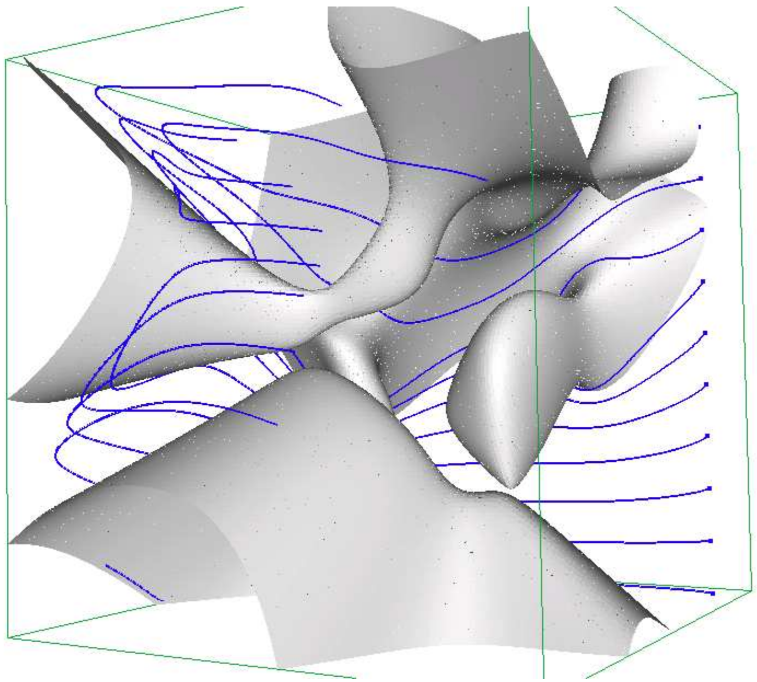
EB AMR Grid Generation Examples: Northern San Francisco Bay



Results: 3D Convergence Study

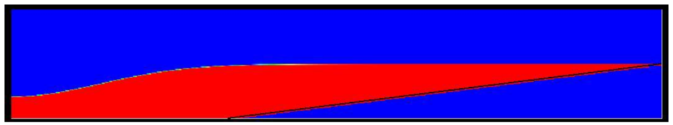
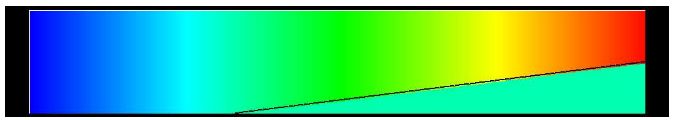
- Below is a 3D convergence study for a $Re = 100$, rotational flow past a complex geometry:

Base Grids	16-32	Rate	32-64
L_1 Norm of U Velocity Error	1.69e-2	**2.32**	3.39e-3
L_2 Norm of U Velocity Error	5.28e-2	**1.76**	1.55e-2
L_1 Norm of W Velocity Error	1.48e-2	**2.29**	3.03e-3
L_2 Norm of W Velocity Error	4.69e-2	**1.83**	1.32e-2



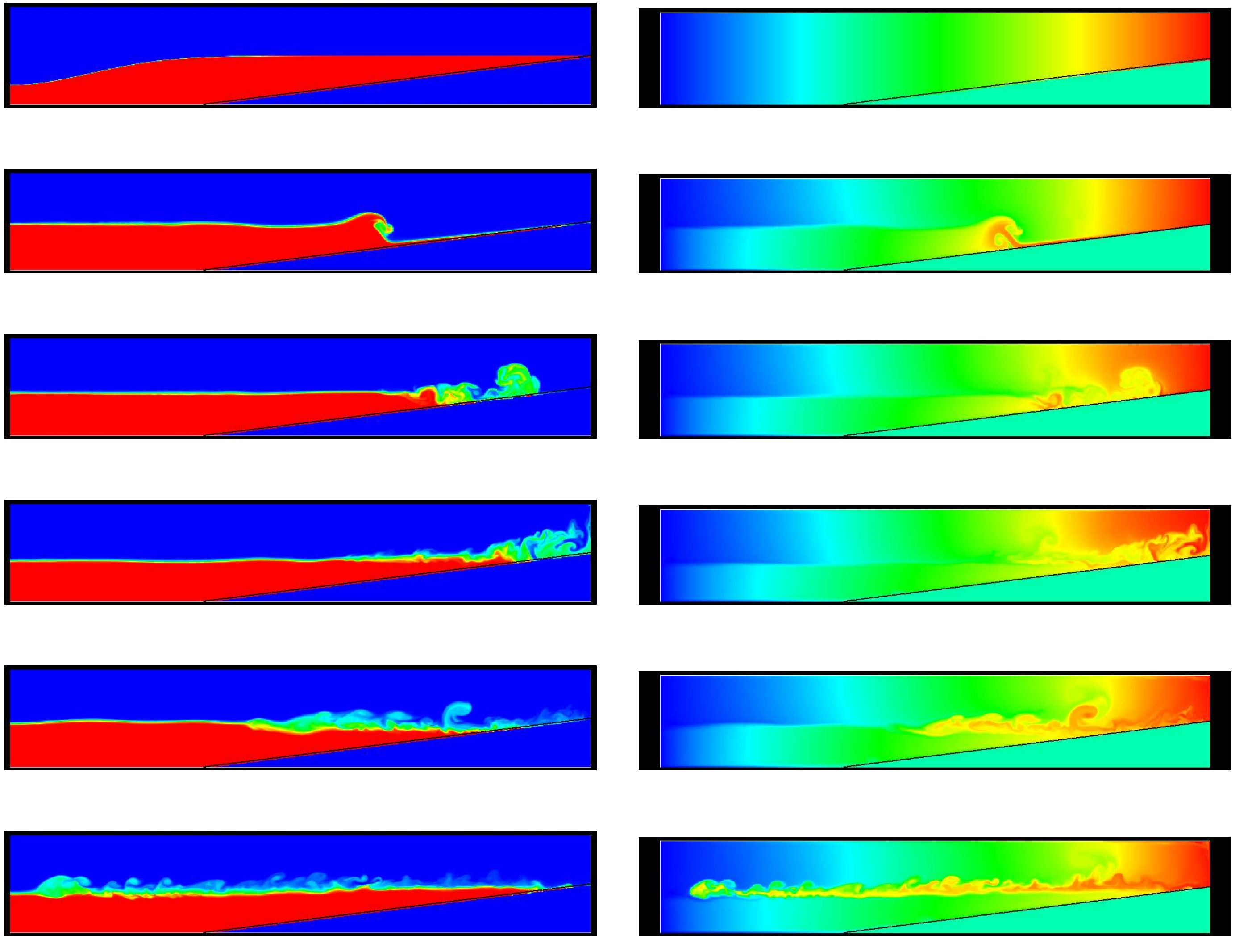
Results: Breaking Internal Waves on a Slope

- Flow is inside a 0.5m tall, by 3m wide tank, with an 8:1 slope starting 1m from the left side
- Below left is the initial density distribution (blue is light fluid, red is heavy fluid), below right is the initial conditions for a passive scalar

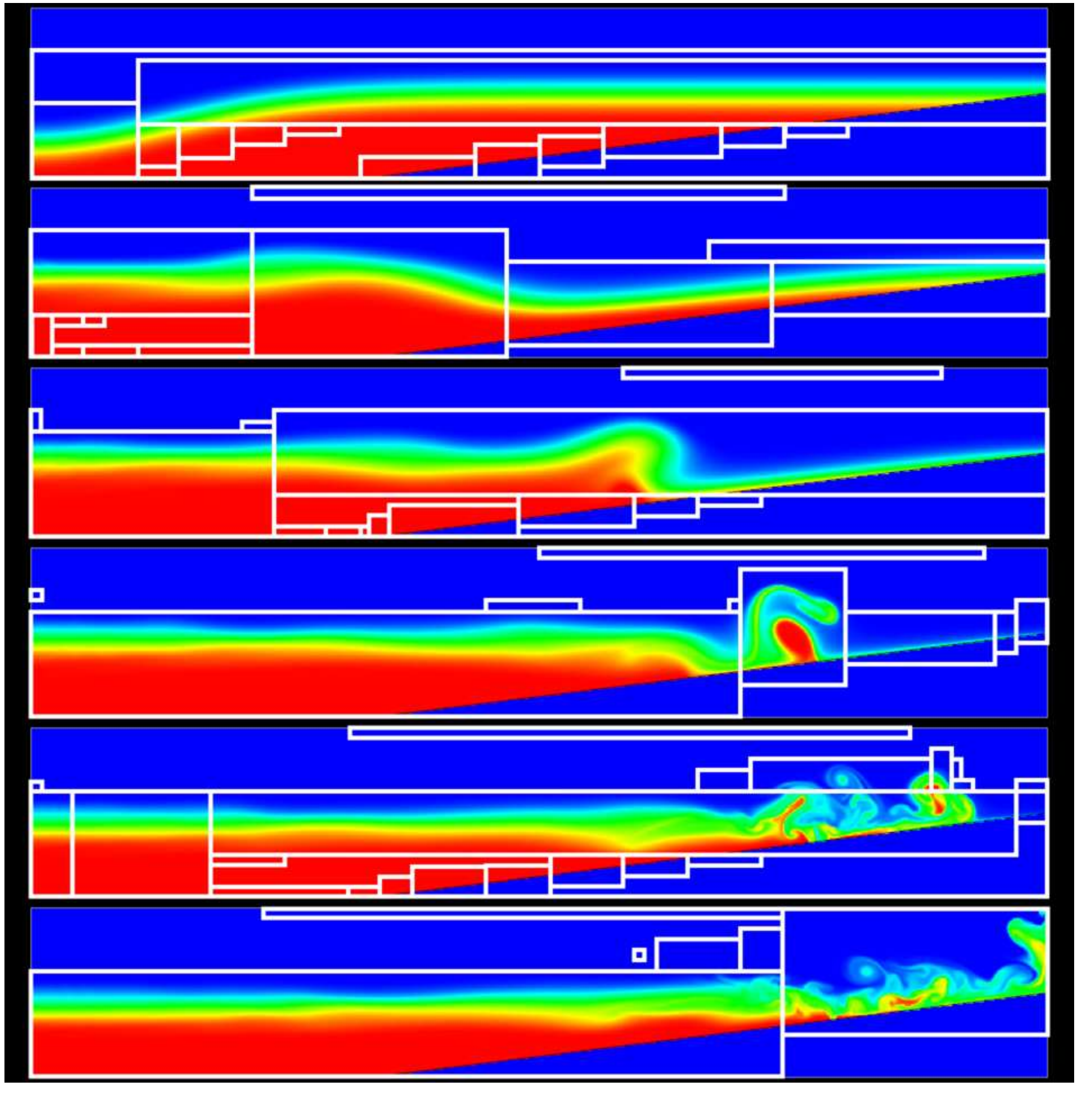



- Density ratio of light fluid to heavy fluid is 1000/1030, and our pycnocline is a step-function. The pycnocline is perturbed on the left side of the tank.
- Thanks to Prof. Fringer of Stanford University for this test problem

Breaking Internal Wave on a Slope (Density left, Scalar right)

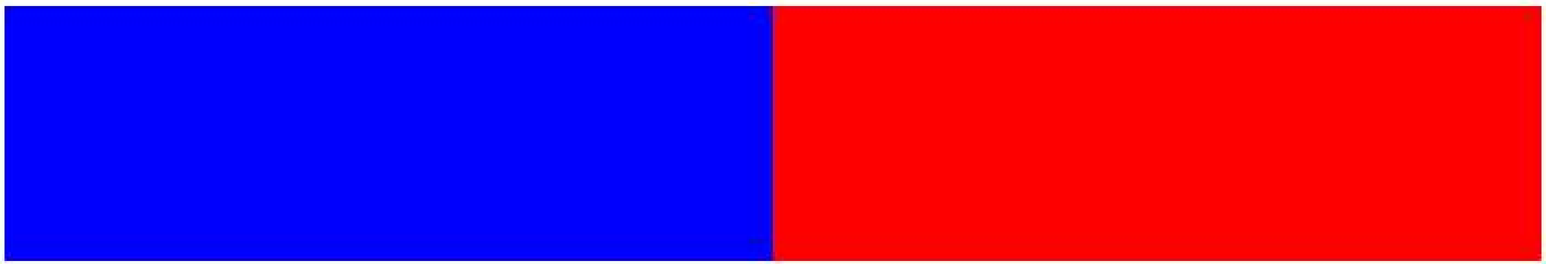


Results: Breaking Internal Waves on a Slope: Smoothed Pycnocline



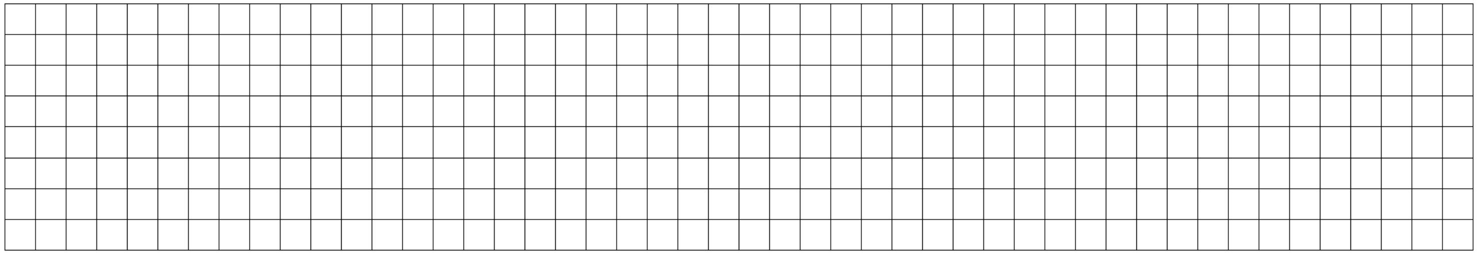
Results: Lock-Exchange with AMR

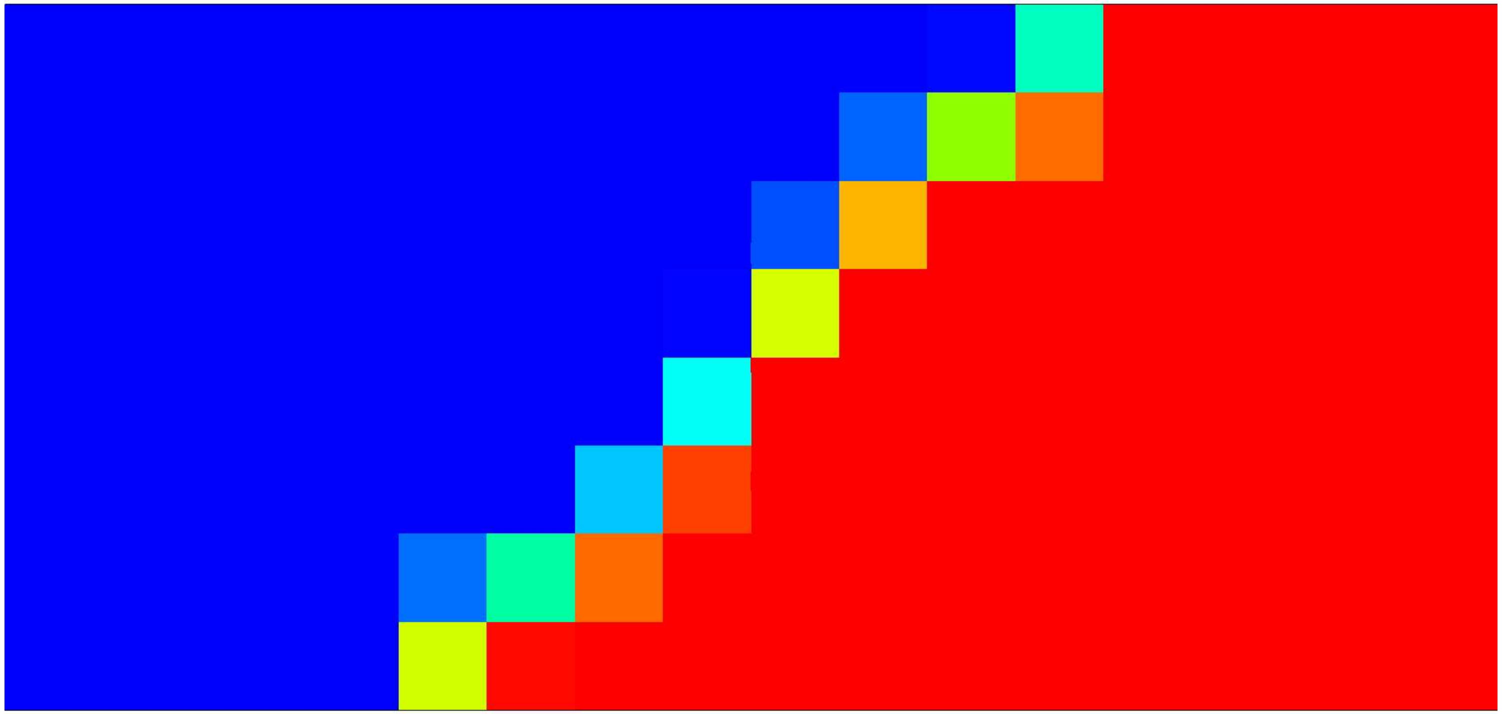
- Flow is inside a 0.5m tall, by 3m wide tank.
- On the left side of the tank we start with light water, on the right is heavy water. The density ratio of light fluid to heavy fluid is 1000/1030.



- On the following lock-exchange slides, the lower figure is a zoom in on the center region of the tank.

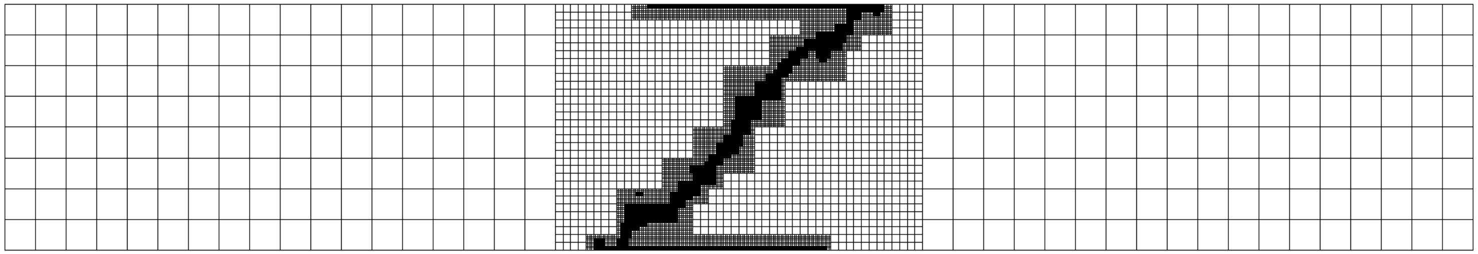
Lock-Exchange: Why is AMR important?

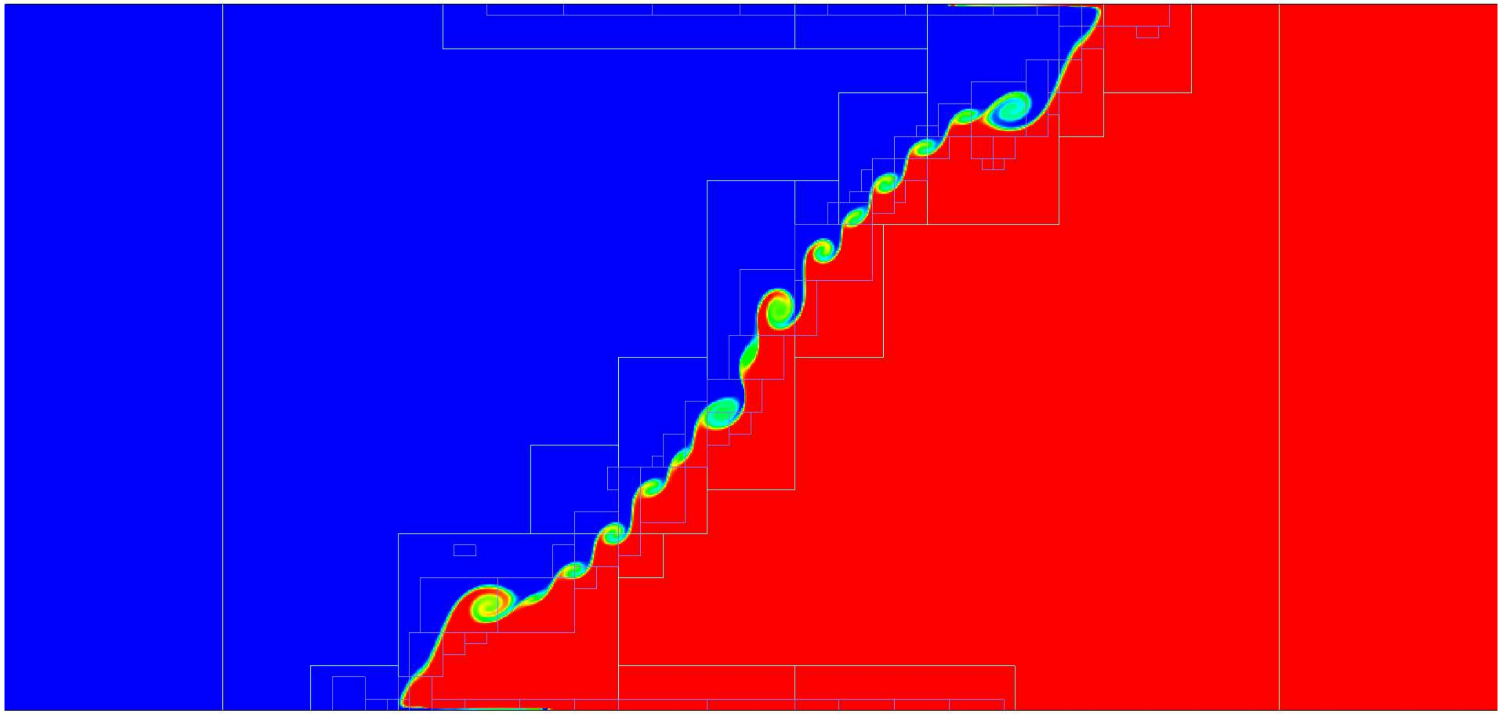




Lock-Exchange: Why is AMR important?

Answer: We can add computational effort only where we need it!





Conclusions and Future Work

- We now have a second-order accurate incompressible Navier-Stokes code that has been validated in 2D & 3D.
- Our AMR version is showing reasonable results and is under review to ensure second-order accuracy.
- Future Work:
  - South China Sea internal waves (with Prof. Fringer)
  - Fourth-order accuracy (with Dr. Colella).

Acknowledgments

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- Check out my web site: http://seesar.lbl.gov/ANAG/staff/barad/